- 11. If the sum of the first 10 terms and the sum of the first 100 terms of a given arithmetic progression are 100 and 10, respectively, then the sum of the first 110 terms is:
- 12. The equations of  $L_1$  and  $L_2$  are y = mx and y = nx, respectively. Suppose  $L_1$  makes twice as large an angle with the horizontal (measured counterclockwise from the positive x-axis) as does  $L_2$ , and that  $L_1$  has 4 times the slope of  $L_2$ . If  $L_1$  is not horizontal, then mn is

(A) 
$$\frac{\sqrt{2}}{2}$$
 (B)  $-\frac{\sqrt{2}}{2}$  (C) 2 (D)  $-2$ 

(E) not uniquely determined by the given information

13. A bug (of negligible size) starts at the origin on the coordinate plane. First it moves 1 unit right to (1, 0). Then it makes a 90° turn counterclockwise and travels  $\frac{1}{2}$  a unit to  $(1, \frac{1}{2})$ . If it continues in this fashion, each time making a 90° turn counterclockwise and traveling half as far as in the previous move, to which of the following points will it come closest?

(A) 
$$(\frac{2}{3}, \frac{2}{3})$$
 (B)  $(\frac{4}{5}, \frac{2}{5})$  (C)  $(\frac{2}{3}, \frac{4}{5})$  (D)  $(\frac{2}{3}, \frac{1}{3})$  (E)  $(\frac{2}{5}, \frac{4}{5})$ 

14. If the function f defined by

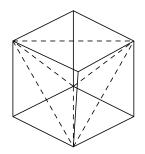
$$f(x)=\frac{cx}{2x+3}, \ x\neq -\frac{3}{2},$$

satisfies f(f(x)) = x for all real numbers x except  $-\frac{3}{2}$ , then c is:

15. A store prices an item in dollars and cents so that when 4% sales tax is added no rounding is necessary because the result is exactly n dollars where n is a positive integer. The smallest value of n is

(A) 1 (B) 13 (C) 25 (D) 26 (E) 100

16. Four of the eight vertices of a cube are vertices of a reglar tetrahedron. Find the ratio of the surface area of the cube to the surface area of the tetrahedron.



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(A) 
$$\sqrt{2}$$
 (B)  $\sqrt{3}$  (C)  $\sqrt{\frac{3}{2}}$  (D)  $\frac{2}{\sqrt{3}}$  (E) 2

- 17. Given that  $i^2 = -1$ , for how many integers n is  $(n+i)^4$  an integer?
- 18. If b > 1,  $\sin x > 0$ ,  $\cos x > 0$  and  $\log_b \sin x = a$ , then  $\log_b \cos x$  equals

(A) 
$$2\log_b(1-b^{\frac{a}{2}})$$
 (B)  $\sqrt{1-a^2}$  (C)  $b^{a^2}$  (D)  $\frac{1}{2}\log_b(1-b^{2a})$ 

(E) none of these

19. Let  $C_1, C_2$  and  $C_3$  be three parallel chords of a circle on the same side of the center. The distance between  $C_1$  and  $C_2$  is the same as the distance between  $C_2$  and  $C_3$ . The lengths of the chords are 20, 16 and 8. The radius of the circle is

(A) 12 (B) 
$$4\sqrt{7}$$
 (C)  $\frac{4\sqrt{65}}{3}$  (D)  $\frac{5\sqrt{22}}{2}$ 

(E) not uniquely determined by the given information

20. A box contains 2 pennies, 4 nickels and 6 dimes. Six coins are drawn without replacement, with each coin having an equal probability of being chosen. What is the probability that the value of the coins drawn is at least 50 cents?

(A) 
$$\frac{37}{924}$$
 (B)  $\frac{91}{924}$  (C)  $\frac{127}{924}$  (D)  $\frac{132}{924}$  (E) none of these